Section 5.1 Fundamental Trig Identities

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2 = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Note that $\sin^2\theta$ represents $(\sin\theta)^2$, $\cos^2\theta$ represents $(\cos\theta)^2$, and so on.

USE LINED PAPER FOR ALL PROBLEMS, IN BOTH NOTES AND HOMEWORK. WE WILL DO SELECT ODD PROBLEMS IN CLASS…YOU WILL DO EVENS FOR HOMEWORK. EACH DAY MAKE SURE YOU KNOW WHICH PROBLEMS YOU NEED TO DO FOR HW BEFORE YOU LEAVE CLASS. SOLID ALGEBRA SKILLS ARE NECESSARY!

In Exercises 9–16, use basic identities to simplify the expression.

9. $\tan x \cos x$

10. $\cot x \tan x$

11. $\sec y \sin(\pi/2 - y)$

12. $\cot u \sin u$

13. $\dfrac{1 + \tan^2 x}{\csc^2 x}$

14. $\dfrac{1 - \cos^2\theta}{\sin\theta}$

15. $\cos x - \cos^3 x$

16. $\dfrac{\sin^2 u + \tan^2 u + \cos^2 u}{\sec u}$

Simplify

24. $\dfrac{1 + \tan x}{1 + \cot x}$

25. $(\sec^2 x + \csc^2 x) - (\tan^2 x + \cot^2 x)$

26. $\dfrac{\sec^2 u - \tan^2 u}{\cos^2 v + \sin^2 v}$

In Exercises 27–32, use the basic identities to change the expression to one involving only sines and cosines. Then simplify to a basic trigonometric function.

27. $(\sin x)(\tan x + \cot x)$

28. $\sin\theta - \tan\theta \cos\theta + \cos(\pi/2 - \theta)$

29. $\sin x \cos x \tan x \sec x \csc x$

30. $\dfrac{(\sec y - \tan y)(\sec y + \tan y)}{\sec y}$

31. $\dfrac{\tan x}{\csc^2 x} + \dfrac{\tan x}{\sec^2 x}$

32. $\dfrac{\sec^2 x \csc x}{\sec^2 x + \csc^2 x}$

In Exercises 33–38, combine the fractions and simplify to a multiple of a power of a basic trigonometric function (e.g., $3\tan^2 x$).

33. $\dfrac{1}{\sin^2 x} + \dfrac{\sec^2 x}{\tan^2 x}$

34. $\dfrac{1}{1 - \sin x} + \dfrac{1}{1 + \sin x}$

35. $\dfrac{\sin x}{\cot^2 x} - \dfrac{\sin x}{\cos^2 x}$

36. $\dfrac{1}{\sec x - 1} - \dfrac{1}{\sec x + 1}$

37. $\dfrac{\sec x}{\sin x} - \dfrac{\sin x}{\cos x}$

38. $\dfrac{\sin x}{1 - \cos x} + \dfrac{1 - \cos x}{\sin x}$

In Exercises 39–46, write each expression in factored form as an algebraic expression of a single trigonometric function (e.g., $(2\sin x + 3)(\sin x - 1)$).

39. $\cos^2 x + 2\cos x + 1$

40. $1 - 2\sin x + \sin^2 x$

41. $1 - 2\sin x + (1 - \cos^2 x)$

42. $\sin x - \cos^2 x - 1$

43. $\cos x - 2\sin^2 x + 1$

44. $\sin^2 x + \dfrac{2}{\csc x} + 1$

45. $4\tan^2 x - \dfrac{4}{\cot x} + \sin x \csc x$

46. $\sec^2 x - \sec x + \tan^2 x$

SOLVING TRIG EQUATIONS

In Exercises 51–56, find all solutions to the equation in the interval $[0, 2\pi)$. You do not need a calculator.

51. $2 \cos x \sin x - \cos x = 0$

52. $\sqrt{2} \tan x \cos x - \tan x = 0$

53. $\tan x \sin^2 x = \tan x$

54. $\sin x \tan^2 x = \sin x$

55. $\tan^2 x = 3$

56. $2 \sin^2 x = 1$

In Exercises 57–62, find all solutions to the equation. You do not need a calculator.

57. $4 \cos^2 x - 4 \cos x + 1 = 0$

58. $2 \sin^2 x + 3 \sin x + 1 = 0$

59. $\sin^2 \theta - 2 \sin \theta = 0$

60. $3 \sin t = 2 \cos^2 t$

61. $\cos(\sin x) = 1$

62. $2 \sin^2 x + 3 \sin x = 2$